

Relativistic Density Dependent Hartree-Fock Approach for Finite Nuclei

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Abstract

The self-energy of the Dirac Brueckner-Hartree-Fock calculation in nuclear matter is parametrized by introducing density-dependent coupling constants of isoscalar mesons in the relativistic Hartree-Fock (RHF) approach where isoscalar meson σ , ω and isovector meson π , ρ contributions are included. The RHF calculations with density dependent coupling constants obtained in this way not only reproduce the nuclear matter saturation properties , but also provide the self-energy with an appropriate density dependence. The relativistic density dependent Hartree-Fock (RDHF) approach contains the features of the relativistic G matrix and in the meantime simplifies the calculation. The ground state properties of spherical nuclei calculated in the RDHF are in good agreement with the experimental data. The contribution of isovector mesons π and ρ , especially the contribution of the tensor coupling of ρ meson , are discussed in this paper.

1 Introduction

It is well known that the relativistic mean field (RMF) theory has been extensively used to investigate the ground state properties of spherical and deformed nuclei[1, 2] in the past several years. Recently , it is used to study the properties of nuclei far from β stability line[3, 4]. Bouyssy et al[5] extended the RMF to the relativistic Hartree-Fock(RHF) approach. The contributions of the exchange terms and isovector mesons in RHF to the ground state properties of nuclei have been emphasized. Since then, the RHF has been developed by several authors[6, 7].

Though the RMF or RHF approach has been quite successful in reproducing the bulk properties of nuclei. However, too large compressibility of nuclear matter is produced in these approaches . It might indicate an incorrect density and momentum behavior of the effective interaction described in the RMF and RHF approaches. Some discrepancies in reproducing the properties of finite nuclei in the standard RMF or RHF calculations are observed. The calculations yield correct binding energies but too little charge radius or vice versa. Such calculations reveal a new "Coester" band in a dependence of $1/r_{ch}$ on E_B which is similar as in the nonrelativistic Brueckner-Hartree-Fock approach (BHF)[9]. A more fundamental and sophisticated way to deal with the many-body problem is the Dirac Brueckner-Hartree-Fock(DBHF) approach[9-12]. Starting from a bare nucleon-nucleon interaction of one boson exchange potential, one solves the Brueckner-Goldstone equation in the nuclear medium. The Brueckner G matrix is the two nucleon effective interaction including nucleon-nucleon short range correlation. The DBHF has been quite successful in reproducing the nuclear matter saturation properties, as well as the compressibility[11, 13]. It provides a correct description of the density- and momentum- dependence of the nucleon self-energy in the nuclear medium. However, the DBHF due to its complexity is mainly restricted to nuclear matter and only very few finite-nuclei calculations are performed so far[14]. Therefore, it is requested to remedy the deficiencies of the RMF or RHF approach with an effective interaction without losing the features of the relativistic G matrix and at same time retain the simplicity.

Recently, there is a growing effort on developing the relativistic effective interaction both in nuclear matter and finite nuclei. Attempts have been also made to improve both binding energy and rms radii of finite nuclei simultaneously in the RMF or RHF with various effec-

tive interactions. Gmuca [15] has parametrized the results of the DBHF in nuclear matter in terms of the RMF with nonlinear scalar and vector self-interactions. Though the coupling constants obtained in such way is density-independent, these mesonic self-interactions implicitly represent the density-dependence. Brockmann and Toki [16] developed a relativistic density dependent Hartree (RDH) approach for finite nuclei. The coupling constants of isoscalar meson σ and ω in the RMF are adjusted at each density by reproducing the nucleon self-energies in nuclear matter resulting from the DBHF instead of fitting them to the empirical nuclear matter saturation properties. The binding energies and rms radii of ^{16}O and ^{40}Ca calculated with the density dependent interaction are in good agreement with experiments. However, Fritz, M  ther and Machleidt [17] pointed out that the Fock terms are not negligible. The relativistic density dependent Hartree-Fork(RDHF) calculations were performed [17]. Due to the uncertainty of the DBHF at very low densities, the extrapolation procedure has to be adopted when applied to the calculation of finite nuclei. The sensitivity of the results calculated by the RDH or RDHF approaches on the extrapolation of the coupling constants at very low densities are observed [16, 17]. However, no detail procedures of the extrapolation were presented there. Since there is no direct restriction on the coupling constants at very low density, in Ref.[18] the scalar and vector potentials of the DBHF results were extrapolated at low densities first by respecting their properties at nuclear matter. Then, the extrapolations of the coupling constants in the RDH or RDHF approaches are restricted by the scalar and vector potentials at the low densities. Therefore, the extrapolations of coupling constants at the low densities in different cases are on an equal level. As mentioned above, the contributions of isovector mesons are neglected in Refs.[16, 17]. It is known that a realistic description of the nucleon-nucleon interaction in terms of meson exchange must include π and ρ . Therefore, it is necessary to develop a relativistic theory for finite nuclei in the RHF with the isovector mesons π , ρ included. Fritz, M  ther [19] discussed the π contribution in the RDHF approach to the bulk properties of finite nuclei. They found that the inclusion of the π -exchange terms in the RDHF slightly improve the agreement between calculation and experiment. On the other hand, Boersma and Malfliet [20] achieved a density dependent parametrization of the Dirac-Brueckner G matrix in nuclear matter which was called an effective DBHF. They used the effective DBHF to systematically analyze a series of spherical nuclei. The results are in good agreement with

the experiments. It should be mentioned that the isovector vector meson ρ was not included in those calculations. In our previous brief report [21], the effects of the isovector ρ meson in the RDHF on the bulk properties for finite nuclei are discussed but ρ tensor coupling was not included. It was found that the Fock exchange terms in the σ - ω model reduce the charge radii, but have less influence on the binding energies. A large repulsion of π contribution at the interior of nuclear is observed. As a result the energy levels of single particles becomes shallow at the presence of π meson. Therefore the total binding energy is reduced and the charge radius is expanded. This effect is partly canceled when the ρ meson is included in the RDHF approach. But the contribution of tensor coupling of ρ meson has not been discussed in these works. In this paper, the effect of tensor coupling of ρ meson is included in the RDHF approach. The systematic study of finite nuclei in terms of the RDHF approach is investigated.

The arrangement of this paper is as follows: The general formalism in this work is presented in Sec.2. The numerical results and main conclusions are included in Sec.3 and Sec.4.

2 The formalism

As in the one-boson-exchange(OBE) description of the NN interaction[22],our starting point is an effective Lagrangian density which couples a nucleon (ψ) to two isoscalar mesons(σ and ω) and two isovector ones(π and ρ) with the following quantum number (J^π, T):

$$\sigma(0^+, 0), \omega(1^-, 0), \pi(0^-, 1), \rho(1^-, 1)$$

and the electromagnetic field (A^μ) is also included.

The effective Lagrangian density can be written as the sum of free and interaction parts:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \tag{1}$$

The free Lagrangian density is given by

$$\mathcal{L}_0 = \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$

$$\begin{aligned}
& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu \cdot \rho^\mu - \frac{1}{4}\mathbf{G}_{\mu\nu} \cdot \mathbf{G}^{\mu\nu} \\
& + \frac{1}{2}(\partial_\mu\pi \cdot \partial^\mu\pi - m_\pi^2\pi^2) - \frac{1}{4}H_{\mu\nu}H^{\mu\nu},
\end{aligned} \tag{2}$$

with

$$F_{\mu\nu} = \partial_\nu\omega_\mu - \partial_\mu\omega_\nu,$$

$$\mathbf{G}_{\mu\nu} = \partial_\nu\rho_\mu - \partial_\mu\rho_\nu,$$

$$H_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu,$$

where the meson fields are denoted by σ , ω_μ , ρ_μ and π , and m_σ , m_ω , m_ρ and m_π are their masses, respectively. The nucleon field is denoted by ψ which has a rest mass M . A_μ is the electromagnetic field. The interaction Lagrangian density is given by

$$\begin{aligned}
\mathcal{L}_I = & g_\sigma\bar{\psi}\sigma\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi - g_\rho\bar{\psi}\gamma_\mu\rho^\mu \cdot \tau\psi \\
& + \frac{f_\rho}{2M}\psi\sigma_{\mu\nu}\partial^\mu\rho^\nu \cdot \tau\psi - e\bar{\psi}\gamma_\mu\frac{1}{2}(1+\tau_3)A^\mu\psi \\
& - \frac{f_\pi}{m_\pi}\psi\gamma_5\gamma_\mu\partial^\mu\pi \cdot \tau\psi,
\end{aligned} \tag{3}$$

here τ and τ_3 are the usual isospin Pauli matrices. The effective strengths of couplings between the mesons and nucleons are denoted by the coupling constants g_i or f_i ($i = \sigma, \omega, \rho, \pi$), respectively. Note that the pseudovector(PV) coupling for π NN interaction is used, because the baryon self-energies are extremely large (about 40 times larger than their PV counterpart) at normal nuclear density if a pseudoscalar coupling is used, which has a drastic effect on the single-particle spectrum[23]. The presence of tensor couplings makes the model Lagrangian density no longer renormalizable and all physical observables should be calculated at the tree level.

2.1 Equations of motion

The equations of motion for the meson fields are easily obtained from the Euler-Lagrange equation

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi)} = 0, \tag{4}$$

with a meson field ϕ . For instance, the σ and ω fields are the solutions of

$$(\square + m_\sigma^2)\sigma = g_\sigma \bar{\psi}\psi, \quad (5)$$

$$(\square + m_\omega^2)\omega_\nu = g_\omega \bar{\psi}\gamma_\nu\psi, \quad (6)$$

with the baryon current conservation $\partial^\mu(\bar{\psi}\gamma_\mu\psi) = 0$. Solving the equation for the meson fields, one then obtains

$$\sigma(x) = g_\sigma \int d^4y D_\sigma(x-y) \bar{\psi}(y)\psi(y), \quad (7)$$

$$\omega^\mu(x) = g_\omega \int d^4y D_\omega^{\mu\nu}(x-y) \bar{\psi}(y)\gamma_\nu\psi(y), \quad (8)$$

where $D_\sigma(x-y)$ and $D_\omega^{\mu\nu}(x-y)$ is the σ and ω meson propagator. Similar expressions are deduced for isovector mesons.

Following standard techniques[24], at the Hartree-Fock level, the average of Hamiltonian at the ground state can be written as

$$\begin{aligned} \langle \Phi_0 | H | \Phi_0 \rangle = & \sum_\alpha \int U_\alpha^\dagger(x) [-i\alpha \cdot \nabla + \gamma_0 m] U_\alpha(x) dx + \int U_\alpha^\dagger(x) \gamma_0 \Sigma_H(x) U_\alpha(x) dx \\ & - \int U_\alpha^\dagger(x) \gamma_0 \int \Sigma_F(x, y) U_\alpha(y) dy dx, \end{aligned} \quad (9)$$

where $U_\alpha(x)$ is nucleon wave function and satisfies the orthogonality relation

$$\int dy U_\alpha^\dagger(y) U_\beta(y) = \delta_{\alpha\beta}$$

Only positive-energy states have been taken into account in the preceding derivation.

2.2 Nuclear matter

Because of the translational and rotational invariance in the rest frame of infinite nuclear matter and the assumed invariance under parity and time reversal, the nucleon self-energy produced by the meson exchanges in nuclear matter can, in general, be written as

$$\Sigma(k_\nu) = \Sigma_s(k_\nu) - \gamma_0 \Sigma_0(k_\nu) + \gamma \cdot \mathbf{k} \Sigma_v(k_\nu) \quad (10)$$

where Σ_s , Σ_0 , Σ_v denote the scalar, time and space components of vector potentials, respectively. In general, they are functions of the four-momentum k_ν of a nucleon and Fermi momentum k_F . Based on the Feynman diagram rules one could derive the nucleon self-energy in nuclear matter. In the RHF approach, the isoscalar mesons in our Lagrangian density give rise to the following contributions to the self-energy[23, 25]:

$$\Sigma_s(k_\nu) = -\left(\frac{g_\sigma}{m_\sigma}\right)^2 \rho_s + \frac{1}{16\pi^2 k} \int_0^{k_F} dq q \hat{M}(q_0) [g_\sigma^2 \Theta_\sigma(k, q) - 4g_\omega^2 \Theta_\omega(k, q)], \quad (11)$$

$$\Sigma_0(k_\nu) = -\left(\frac{g_\omega}{m_\omega}\right)^2 \rho_B - \frac{1}{16\pi^2 k} \int_0^{k_F} dq q [g_\sigma^2 \Theta_\sigma(k, q) + 2g_\omega^2 \Theta_\omega(k, q)], \quad (12)$$

$$\Sigma_v(k_\nu) = -\frac{1}{(8\pi^2 k^2)} \int_0^{k_F} dq q \hat{Q}(q_\nu) [g_\sigma^2 \Phi_\sigma(k, q) + 2g_\omega^2 \Phi_\omega(k, q)], \quad (13)$$

where

$$\begin{aligned} \hat{M}(q_\nu) &= \frac{M^*(q_\nu)}{q_0^*(q_\nu)}, & \hat{Q}(q_\nu) &= \frac{q^*(q_\nu)}{q_0^*(q_\nu)} \\ \Theta_i(k, q) &= \ln \left| \frac{(k+q)^2 + m_i^2}{(k-q)^2 + m_i^2} \right| \\ \Phi_i(k, q) &= \frac{k^2 + q^2 + m_i^2}{4kq} \Theta_i(k, q) - 1, & i &= \sigma, \omega, \rho, \pi \\ \mathbf{k}^* &= \mathbf{k}(1 + \Sigma_v(k_\nu)), & k^* &= |\mathbf{k}^*| \\ M^* &= M + \Sigma_s(k_\nu), \\ k_0^* &= k_0 + \Sigma_0(k_\nu) = (k^{*2} + M^{*2})^{1/2}. \end{aligned}$$

The scalar and vector densities are

$$\begin{aligned} \rho_s &= \frac{2}{\pi^2} \int_0^{k_F} q^2 \hat{M}(q) dq, \\ \rho_B &= \frac{2}{3\pi^2} k_F^3 \end{aligned} \quad (14)$$

The contributions of the isovector mesons π and ρ to the self-energy is given in the Appendix A. The first terms of the Σ_s and Σ_0 are the Hartree terms, which are energy-independent. The rest terms are the Fock terms, which are almost $1/k$ energy dependent. The coupled nonlinear integral equations (11-13) have to be solved self-consistently.

With these nucleon self-energies, following the procedure discussed in Sec3.1, we can obtain the coupling constants g_σ and g_ω at each baryon density.

2.3 Finite nuclei

At the case of spherical, closed-subshell nuclei, a single-particle baryon state with energy E_α is specified by the set of quantum numbers

$$\alpha = (q_\alpha, n_\alpha, l_\alpha, j_\alpha, m_\alpha) \equiv (a, m_\alpha)$$

where $q_\alpha = -1$ for a neutron state and $q_\alpha = +1$ for a proton state. The nucleon wave function can be written as

$$U_\alpha(x) = \frac{1}{r} \begin{pmatrix} iG_a(r) \\ F_a(r)\sigma \cdot \hat{\mathbf{r}} \end{pmatrix} \mathcal{Y}_\alpha \chi_{1/2}(q_\alpha), \quad (15)$$

where $\chi_{1/2}(q_\alpha)$ is an isospinor, and the angular and spin parts of the nucleon spinor can be written as

$$\mathcal{Y}_\alpha(\hat{\mathbf{r}}) = \sum_{\mu_\alpha, s_\alpha} \langle l_\alpha \frac{1}{2} \mu_\alpha s_\alpha | j_\alpha m_\alpha \rangle Y_{l_\alpha}^{\mu_\alpha}(\hat{\mathbf{r}}) \chi_{1/2}(s_\alpha).$$

The spinors $U_\alpha(r)$ are normalized according to

$$\int d^3\mathbf{r} U_\alpha^\dagger(x) U_\alpha(x) = \int_0^\infty [G_a^2(r) + F_a^2(r)] dr = 1.$$

The HF solution is obtained by requiring that the total binding energy

$$E = \langle \phi_0 | H | \phi_0 \rangle - AM,$$

is stationary with respect to variations of the spinors U_α (i.e. of G_α and F_α) such that the normalization relation is preserved

$$\delta[E - \sum_{\alpha(occ)} E_\alpha \int U_\alpha^\dagger(r) U_\alpha(r) d^3r] = 0 \quad (16)$$

using the expression of nucleon spinor, after a lengthy derivation, the HF equations for the self-consistent wave functions (G_α, F_α) and energies E_α will be obtained. The radial Dirac equation take the following form:

$$\frac{d}{dr} \begin{pmatrix} G_a(r) \\ F_a(r) \end{pmatrix} = \begin{pmatrix} -\frac{\kappa_\alpha}{r} - \Sigma_{T,a}^D(r) & M + E_\alpha + \Sigma_{S,a}^D(r) - \Sigma_{0,a}^D \\ M - E_\alpha + \Sigma_{S,a}^D(r) + \Sigma_{0,a}^D & \frac{\kappa_\alpha}{r} + \Sigma_{T,a}^D(r) \end{pmatrix} \begin{pmatrix} G_a(r) \\ F_a(r) \end{pmatrix}$$

$$+ \begin{pmatrix} -X_\alpha(r) \\ Y_\alpha(r) \end{pmatrix} \quad (17)$$

here, $\Sigma_{S,a}^D$, $\Sigma_{0,a}^D$ and $\Sigma_{T,a}^D$ are direct contributions to the self-energy and can be written as

$$\Sigma_{T,a}^D(r) = [\Sigma_\rho^T(r) + \Sigma_\rho^{VT(1)}(r)]q_\alpha \quad (18)$$

$$\Sigma_{S,a}^D(r) = \Sigma_\sigma \quad (19)$$

$$\Sigma_{0,a}^D(r) = \Sigma_\omega(r) + [\Sigma_\rho^S(r) + \Sigma_\rho^{VT(2)}(r)]q_\alpha + \frac{1}{2}(1 + q_\alpha)\Sigma_c(r) \quad (20)$$

whereas X_α and Y_α come from exchange (Fock) contribution. The quantity κ_α is $(2j_\alpha + 1)(l_\alpha - j_\alpha)$.

In this work we consider nuclei with a closed proton and neutron shell only, therefore, the isovector pseudoscalar meson yield no contributions in the Hartree approximation. The Hartree contributions come from σ , ω , ρ mesons and Coulomb force are given as

$$\Sigma_\sigma(r) = -g_\sigma(\rho_B(r))m_\sigma \int_0^\infty g_\sigma(\rho_B(r'))\rho_S(r')\tilde{I}_0(m_\sigma r_<)\tilde{K}_0(m_\sigma r_>)r'^2 dr' \quad (21)$$

$$\Sigma_\omega(r) = g_\omega(\rho_B(r))m_\omega \int_0^\infty g_\omega(\rho_B(r'))\rho_B(r')\tilde{I}_0(m_\omega r_<)\tilde{K}_0(m_\omega r_>)r'^2 dr' \quad (22)$$

$$\Sigma_\rho^S(r) = g_\rho^2 m_\rho \int_0^\infty [\rho_{B,p}(r') - \rho_{B,n}(r')]\tilde{I}_0(m_\rho r_<)\tilde{K}_0(m_\rho r_>)r'^2 dr' \quad (23)$$

$$\Sigma_\rho^{VT(1)} = -\frac{f_\rho}{2mg_\rho} \frac{d}{dr} \Sigma_\rho^S(r) \quad (24)$$

$$\Sigma_\rho^{VT(2)}(r) = -\frac{g_\rho f_\rho}{2M} m_\rho \int_0^\infty [\rho_{T,p}(r') - \rho_{T,n}(r')]\left[\frac{d}{dr}\tilde{I}_0(m_\rho r_<)\tilde{K}_0(m_\rho r_>)\right]r'^2 dr' \quad (25)$$

$$\begin{aligned} \Sigma_\rho^T(r) = & -\left(\frac{f_\rho}{2M}\right)^2 \{m_\rho^3 \int_0^\infty [\rho_{T,p}(r') - \rho_{T,n}(r')]\tilde{I}_1(m_\rho r_<)\tilde{K}_1(m_\rho r_>)r'^2 dr' \\ & - [\rho_{T,p}(r) - \rho_{T,n}(r)]\} \end{aligned} \quad (26)$$

with the definitions

$$\rho_{S,norp} = \frac{1}{4\pi r^2} \sum_{b(norp)} \hat{j}_b^2 [G_b^2(r) - F_b^2(r)]$$

$$\rho_{B,norp} = \frac{1}{4\pi r^2} \sum_{b(norp)} \hat{j}_b^2 [G_b^2(r) + F_b^2(r)]$$

$$\rho_{T,norp} = \frac{1}{4\pi r^2} \sum_{b(norp)} \hat{j}_b^2 [2G_b(r)F_b(r)]$$

and $r_<$ ($r_>$) is the smaller (larger) of the r' and r .

The functions $\tilde{I}_L(x)$ and $\tilde{K}_L(x)$ arise from the multiple expansion of the meson propagator in the coordinate space and are defined using the modified spherical Bessel functions of the first and third kind I and K:

$$\tilde{I}_L(x) = \frac{I_{L+\frac{1}{2}}(x)}{\sqrt{x}}, \quad \tilde{K}_L(x) = \frac{K_{L+\frac{1}{2}}(x)}{\sqrt{x}}$$

The explicit expressions of exchange contributions X_α and Y_α are given in the Appendix B.

3 Results and Discussions

3.1 Parameterization of DBHF

The self-energy obtained in the RHF is of very weak energy-dependence and too strong density-dependence due to the fact that the short range correlation has not been considered . Attempts to incorporate the effects of the short range correlation described in the DBHF approach have been made by introducing density- and momentum-dependent effective coupling constants of mesons in the RHF approach . In order to make comparison with the DBHF results, the scalar and vector potentials can be obtained by

$$U_s(k, k_F) = \frac{\Sigma_s - M\Sigma_v}{1 + \Sigma_v}, \quad U_o(k, k_F) = \frac{-\Sigma_o + E_k\Sigma_v}{1 + \Sigma_v}, \quad (27)$$

where $E_k = \sqrt{k^{*2} + M^{*2}} - \Sigma_o$. The momentum dependence of the potentials usually are relatively weak and neglected in the description of ground state properties of finite nuclei . Therefore , the momentum average within the Fermi sea is performed,

$$U_{s(o)}(k_F) = \frac{\int_0^{k_F} k^2 U_{s(o)}(k, k_F) dk}{\int_0^{k_F} k^2 dk}. \quad (28)$$

At very low density of nuclear matter the DBHF results are not reliable and remain unknown. Therefore, the extrapolation of the coupling constants outside the density points in

the DBHF has to be done when they are applied to the calculation of finite nuclei. In order to remove the sensitivity, the extrapolation of scalar and vector potentials U_s and U_o of the DBHF results at low densities are done by setting $U_s = 0$, $U_o = 0$ at $\rho = 0$. It is known that the scalar and vector potentials in RMF or RHF are almost linear dependent on the density. Due to the two-body correlation the scalar and vector potentials approach to zero smoothly as the density goes to zero. A polynomial fit of the scalar and vector potentials with respect to the density are performed. The extrapolation and interpolation of U_s and U_o are shown in Fig.1, where the circles are the DBHF results in nuclear matter using the Bonn A potential[13]. The density dependence of the coupling constants are then adjusted in the cases of RMF or RHF with or without isovector mesons to reproduce the nucleon self-energies at each densities resulting from the DBHF. The nucleon and σ and ω meson masses are chosen to be the same as the DBHF calculation, where $M = 938.9$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 782.6$ MeV. The pseudo-vector coupling for πNN and vector and tensor coupling for ρNN are adopted. The masses and coupling constants of isovector mesons are fixed to be $m_\pi = 138$ MeV, $m_\rho = 770$ MeV, $\frac{f_\pi^2}{4\pi} = 0.08$, $\frac{g_\rho^2}{4\pi} = 0.55$, $\frac{f_\rho}{g_\rho} = 3.7$ [5]. The density dependence of coupling constants in different cases are shown in Fig.2. The presence of the pion introduces a large repulsive force, so the scalar coupling constant becomes larger and the vector coupling constant gets smaller to balance the repulsive force, especially at normal and high densities. However, the pion contribution is partly canceled by the presence of vector part of ρ meson at the symmetric nuclear matter. The tensor coupling of ρ meson has large effect at high density. As a result, the coupling constant of σ meson becomes larger than that of ω meson at high densities. The results obtained in this paper are somewhat different from those in ref.[19]. The reason for this discrepancy is that the zero-range components of the pion-exchange were removed there. The cases with and without the contact interactions have been discussed in more detail in ref.[6]. It is found in our calculations that the effects of the contact interactions mainly cause a renormalization in g_σ and g_ω coupling constants. The removal of the zero-range components of the pion and rho exchange would increase the binding energy and reduce the charge r.m.s radius. No qualitative improvement has been found. Therefore, only the cases with the zero-range components of the π and ρ exchange are presented in this paper.

3.2 The ground state properties of finite nuclei

The ground state properties of four stable doubly closed-shell nuclei ^{16}O , ^{40}Ca , ^{48}Ca and ^{90}Zr are calculated with these density dependent coupling constants in the RDH and RDHF approaches. The set of coupled differential equations(17) is solved in the coordinate space following the method of Ref.[5]. The self-consistence is achieved by an iterative procedure. It is different from a matrix diagonalization method adopted in Refs.[16, 17], where a termination of a complete set of bases has been performed both for baryon and meson results. Our computer code has been carefully checked with the results of ref.[5, 27].

In order to investigate the effect of the density-dependence, the results for ^{16}O and ^{40}Ca obtained in the RMF and RHF with $\sigma + \omega$ (RHF1), $\sigma + \omega + \pi$ (RHF2) and $\sigma + \omega + \pi + \rho$ (RHF3) are list in Table 1. The coupling constants are determinated to reproduce the DBHF results in nuclear matter (OBE potential A) at saturation density $K_F = 1.40 \text{ fm}^{-1}$. It should be mentioned that the results are different from those of the usual RMF and RHF calculations, where the coupling constants and scalar meson mass are adjusted to reproduce the empirical saturation properties of nuclear matter as well as the rms charge radius of ^{40}Ca . Because of relative large saturation density obtained in the DBHF and the scalar meson mass $m_\sigma = 550 \text{ Mev}$ adopted in this calculation, the binding energies and rms charge radii calculated here are both much smaller than the experimental data. However, the main purpose of Table 1 is to show the difference in various cases mentioned above as well as the effect of the density-dependence in comparison with Table 2. The calculations with density-dependent effective interaction are performed, where the coupling constants at each baryon density come from the parametrization of the DBHF result in nuclear matter as discussed in section 3.1. Various cases, RDH, RDHF with $\sigma + \omega$ (RDHF1), $\sigma + \omega + \pi$ (RDHF2) and $\sigma + \omega + \pi + \rho$ (RDHF3) are investigated and the results for the nuclei ^{16}O , ^{40}Ca , ^{48}Ca and ^{90}Zr are displayed in Table 2. In order to investigate the sensitivity of the ρ meson coupling constant, two values of ρ coupling constants are adopted in the calculations: $\frac{g_\rho^2}{4\pi} = 0.55$ (RDHF3A) and 0.99 (RDHF3B) without tensor coupling. The results with ρ NN tensor coupling ($\frac{g_\rho^2}{4\pi} = 0.55$, $\frac{f_\rho}{g_\rho} = 3.7$) are given as RDHF3C.

The importance of the density dependent approaches is clearly demonstrated in Table 1 and Table 2. The calculations in either RMF or RHF with constant coupling constants

	RMF	RHF1	RHF2	RHF3
¹⁶ O				
$E_B/A(\text{Mev})$	-5.62	-6.02	-4.86	-5.67
$r_{ch}(\text{fm})$	2.48	2.39	2.53	2.57
$1s_{1/2}(\text{Mev})$	44.34	45.08	40.78	43.21
$1p_{3/2}(\text{Mev})$	18.96	21.15	17.50	18.51
$1p_{1/2}(\text{Mev})$	9.62	7.85	9.49	10.98
⁴⁰ Ca				
$E_B/A(\text{Mev})$	-6.36	-6.69	-5.84	-6.38
$r_{ch}(\text{fm})$	3.14	3.04	3.16	3.22
$1d_{5/2}(\text{Mev})$	16.54	18.61	15.72	16.35
$2s_{1/2}(\text{Mev})$	7.07	5.60	7.98	8.81
$1d_{3/2}(\text{Mev})$	6.92	5.70	6.50	7.79

Table 1: Ground state properties of ¹⁶O and ⁴⁰Ca calculated by RHA and RHF. The binding energy per nucleon E_B/A , the charge rms radius r_c and single-particle energies of proton states.

	RDH	RDHF1	RDHF2	RDHF3A	RDHF3B	RDHF3C	Exp.
¹⁶ O							
$E_B/A(\text{Mev})$	-7.44	-7.48	-6.96	-7.29	-7.57	-7.41	-7.98
$r_{ch}(\text{fm})$	2.59	2.50	2.64	2.61	2.59	2.68	2.73
$1s_{1/2}(\text{Mev})$	43.97	43.87	41.11	42.76	44.13	42.98	40 ± 8
$1p_{3/2}(\text{Mev})$	21.77	23.60	21.11	21.85	22.47	21.31	18.4
$1p_{1/2}(\text{Mev})$	16.16	16.08	15.60	16.07	16.47	15.72	12.1
⁴⁰ Ca							
$E_B/A(\text{Mev})$	-7.88	-7.91	-7.49	-7.74	-7.94	-7.81	-8.55
$r_{ch}(\text{fm})$	3.26	3.17	3.29	3.27	3.26	3.35	3.48
$1d_{5/2}(\text{Mev})$	19.27	21.17	19.08	19.52	19.89	18.95	15.5
$2s_{1/2}(\text{Mev})$	13.69	14.08	14.20	14.19	14.14	13.78	10.9
$1d_{3/2}(\text{Mev})$	13.29	13.48	12.77	13.09	13.35	12.67	8.3
⁴⁸ Ca							
$E_B/A(\text{Mev})$	-8.02	-7.96	-7.45	-7.59	-7.70	-7.60	-8.67
$r_{ch}(\text{fm})$	3.27	3.17	3.30	3.28	3.27	3.37	3.47
$1d_{5/2}(\text{Mev})$	24.35	29.17	24.34	26.37	27.96	26.72	20.0
$2s_{1/2}(\text{Mev})$	17.25	19.77	19.79	20.90	21.77	20.61	15.8
$1d_{3/2}(\text{Mev})$	18.68	21.79	23.05	24.92	26.38	23.53	15.3
⁹⁰ Zr							
$E_B/A(\text{Mev})$	-7.94	-7.92	-7.55	-7.67	-7.78	-7.68	-8.71
$r_{ch}(\text{fm})$	4.00	3.89	4.02	4.00	3.99	4.10	4.27
$2p_{3/2}(\text{Mev})$	11.01	13.09	12.86	13.61	14.19	13.52	11.0
$1f_{5/2}(\text{Mev})$	13.14	15.61	16.67	18.12	19.24	16.93	12.3
$2p_{1/2}(\text{Mev})$	9.42	11.16	11.77	12.49	13.05	11.91	9.5

Table 2: Ground state properties of ¹⁶O , ⁴⁰Ca , ⁴⁸Ca and ⁹⁰Zr calculated by RDH and RDHF. The binding energy per nucleon E_B/A , the charge rms radius r_c and single-particle energies of proton states.

produce much smaller binding energies of nucleon in comparison with the experiments. In contrast, the density-dependent interactions increase both binding energy and charge radius, which imply the removal from the so-called Coester band[8]. The results in the relativistic density-dependent approaches are largely improved and closer to the experimental values. The slight differences from ref.[16, 17] are due to the different extrapolation procedures. The Fock exchange term in the σ - ω model reduces the charge radii, but has less influence on the binding energies. A large repulsion of pion contribution at the interior of nucleus is found. As a result, the energy levels of single particles become shallow at the presence of pion. Therefore, the total binding energy is reduced and the charge radius is expanded. This effect is partly canceled by the ρ meson exchange contribution. The charge radii of nuclei calculated in the RDHF with isovector mesons are much close to those obtained in the RDH, which can also be observed in the charge density distributions. In comparison of the RDHF3A and RDHF3B, the results are not sensitive to the strength of the ρ meson coupling. The binding energies for the strong coupling constant of ρ meson $\frac{g_\rho^2}{4\pi} = 0.99$ is about 2% bigger than those for $\frac{g_\rho^2}{4\pi} = 0.55$ and the rms radius is reduced less than 1%. With ρ tensor coupling, it can be found that the results of both binding energy and rms charge radius are improved. The binding energies of nuclei is increased slightly, but the charge radii of nuclei are improved largely in comparison with experiments.

Figure 3 shows the density distribution of nuclei. The dash-dotted curves are the results of the RHF3, corresponding to the fourth column in Table 1. The dashed, dotted and solid ones correspond to those of the RDH and RDHF without and with isovector mesons (RDHF1, RDHF3A, RDHF3C), respectively. It is found that the charge densities are reduced at the nuclear interior and have long tail due to the relative strong coupling constants at the nuclear surface in the density dependent calculations. The Fock contribution of σ and ω in the RDHF produce a squeezing effect and give a large central density. The repulsive contribution of π reduces the interior density and the results of the RDHF3A are very close to those of the RDH. Though the density at center is still higher than the experimental results, it is a parameter-free calculation in the sense that no parameters are adjusted for the calculations of the many-body problem. However, the results with density dependent coupling constants are in a reasonable good agreement with the experiments.

As well known, nonrelativistic BHF calculations with various two-body nucleon-nucleon

potentials, such as Reid soft-core ,Hamada-Johnson potentials , reveal a "Coester" band in dependence of $1/r_c$ on E_B/A . In the RMF or RHF calculations, the dependence of $1/r_c$ on E_B/A at a fix K_F , with variation of the scalar meson mass and therefore the variation of the coupling constants , formed a new "Coester" band. A better estimate of the merits of the present work can be expected upon the comparison with the BHF "Coester" band and RHF "Coester" band. Those "Coester" bands are plot in Fig.4 for ^{16}O and ^{40}Ca , the dash-dotted line indicates the BHF "Coester" band, which is taken from a so called generalized BHF calculations by Kümmel et al.[9]. The dotted and dashed lines represent the RHF results , which are obtained in the RHF3 by varying the scalar meson mass as well as the different coupling constants to reproduce the nuclear matter saturation properties at $K_F = 1.40\text{fm}^{-1}$ (1) and $K_F = 1.30\text{fm}^{-1}$ (2) resulting from the DBHF approach , respectively . The results in the case of the RDHF3C are displayed by a solid line in the figure. The density dependent approach forms a new line away from all of the "Coester" band of conventional BHF and seems to be much closer to the experimental values than the BHF and RHF calculations.

The spin-orbit splittings of nuclei are given in Table 3. It can be seen that the spin-orbit splitting in the RMF and RHF approaches is larger than the experimental data, which indicates the larger spin-orbit force. It is known that the spin-orbit force is related to the derivative of the potentials with respect to the space and is a surface effect. The density dependent approaches reduce the sharp surface and, therefore, reduce the spin-orbit splitting. The Fock terms of the σ and ω exchange increase the spin-orbit splitting, while the π and ρ exchanges give the opposite contribution. A large reduction of the spin-orbit splitting due to pion-exchange is found, especially for heavy nuclei with large neutron excess. A flip of the spin-orbit splitting in ^{208}Pb is observed in the calculation of RDHF with all mesons included, which is certainly not physical. It might indicate that the free coupling constants of the isovector mesons adopted in the RDHF are too strong for the nuclear structure calculation. A similar observation was obtained in the calculation of the relativistic optical potential in the RHF approach[25]. The density dependent coupling constants for isovector mesons may also be required.

It is known that the single particle densities are not directly provided by experiments . The only way to gain some insight in the single particle distribution is to study the difference between density distribution of nearby nuclei. In Fig.5 ,we give the charge distribution

Nuclei	RMF	RHF1	RHF3	RDH	RDHF1	RDHF3A	RDHF3C	Exp.
^{16}O	9.34	13.30	7.53	5.61	7.52	5.78	5.60	6.3
^{40}Ca	9.62	12.82	8.56	5.78	7.69	6.43	6.28	7.2
^{48}Ca	9.32	12.52	5.32	5.67	7.37	1.45	3.19	4.3

Table 3: Spin-orbit splittings of protons for the $1p$ shell in ^{16}O and the $1d$ shell in ^{40}Ca and ^{48}Ca .

Nuclei	RHF3	DBHF	RDHF1	RDHF3C	Exp.
^{16}O	-0.03	-0.03	-0.03	-0.02	-0.02
^{40}Ca	-0.05	-0.06	-0.05	-0.04	-0.07 - 0.10
^{48}Ca	0.23	0.13	0.15	0.22	0.16 - 0.23
^{90}Zr	0.11	0.04	0.07	0.11	0.07

Table 4: Neutron skin thickness $\Delta_{np} = r_n - r_p$ for ^{16}O , ^{40}Ca , ^{48}Ca , ^{90}Zr . The DBHF results are taken from [20].

difference between ^{40}Ca and ^{48}Ca , multiplied by r^2 . The difference of the neutron densities between ^{40}Ca and ^{48}Ca are plotted in Fig.6 . The shaded area presents the experimental data, the dashed curve is the results of the RDH , the dotted one the results of RDHF1 , and the solid one is obtained in the case of RDHF3C. It can be seen that the results obtained in the RDHF3C are superior to those of the RDH and RDHF1 in comparison with experiments. It means that the isospin dependence can not be correctly described by the RDH as well as RDHF1 without including isovector mesons.

Neutron skin thickness is an important quantity to study isotope shifts. It is defined as the difference between neutron and proton rms radii: $\Delta_{np} = r_n - r_p$. The neutron skin thickness of ^{16}O , ^{40}Ca , ^{48}Ca and ^{90}Zr are given in Table 4 , and the Δ_{np} versus the asymmetry parameter $(N-Z)/A$ for ^{40}Ca , ^{48}Ca and ^{90}Zr are shown in Fig.7 . The results of the RDHF3C seem to be similar to those of the RMF , which are different from what obtained in ref.[20] . More information of of isospin dependence the ground state properties are required .

4 Conclusion

In summary , the RDH and RDHF approaches with the density-dependent effective coupling constants of isoscalar mesons can incorporate the DBHF results and contain the nucleon-nucleon correlation effects . Inclusion of the NN correlation let to a substantial improvement in the microscopic description of bulk properties of nuclei . The Fork exchange terms are not negligible , though the exchange contributions are relatively weak than those of the Hartree direct term in the relativistic approach and their contribution to the binding energy may be compensated by the variation of the coupling constants . The important contributions from the isovector meson π and as well as , to some extent , ρ meson are not included in the mean field approach . It is found that the isovector mesons π and ρ play an important role in the spin-orbit splitting as well as the isospin dependent quantities . The tensor coupling of ρ meson gives a constructive contribution to the bind energy , especially the rms charge radius , therefore improves the agreement with the experimental data . More information of the isospin denpendence of nuclear properies is required to provide constraints on the coupling constants of isovector mesons π and ρ in the nuclear medium .

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Appendix A. Self-Energy in Nuclear Matter

Base on Hartree-Fork approach , the nucleon self-energy in nuclear matter coming from the contributions of isovector pseudoscalar meson π can be written as follow:

$$\Sigma_s^\pi(k) = \frac{3}{8\pi^2 k} \int_0^{k_F} dq q \left(\frac{f_\pi}{m_\pi}\right)^2 \hat{M}(2kq - \frac{1}{2}m_\pi^2 \Theta_\pi) \quad (A.1)$$

$$\Sigma_0^\pi(k) = \frac{3}{8\pi^2 k} \int_0^{k_F} dq q \left(\frac{f_\pi}{m_\pi}\right)^2 [-\frac{1}{2}m_\pi^2 \Theta_\pi + 2kq] \quad (A.2)$$

$$\Sigma_v^\pi(k) = -\frac{3}{8\pi^2 k^2} \int_0^{k_F} dq q \left(\frac{f_\pi}{m_\pi}\right)^2 [q\hat{Q}k\Theta_\pi - \hat{Q}(k^2 + q^2)\Phi_\pi] \quad (A.3)$$

The contribution coming from isovector vector meson ρ can be write as :

$$\begin{aligned} \Sigma_s^\rho(k) = \frac{3}{8\pi^2 k} \int_0^{k_F} dq q \{ & -2g_\rho^2 \hat{M}\Theta_\rho + 3\left(\frac{f_\rho}{2M}\right)^2 \hat{M}(2kq - \frac{1}{2}m_\rho^2 \Theta_\rho) \\ & + 3g_\rho \left(\frac{f_\rho}{2M}\right) [q\hat{Q}\Theta_\rho - 2k\hat{Q}\Phi_\rho] \} \end{aligned} \quad (A.4)$$

$$\Sigma_0^\rho(k) = \frac{3}{8\pi^2 k} \int_0^{k_F} dq q \{ g_\rho^2 \Theta_\rho + \left(\frac{f_\rho}{2M}\right)^2 [2kq - \frac{1}{2}m_\rho^2 \Theta_\rho] \} \quad (A.5)$$

$$\begin{aligned} \Sigma_v^\rho(k) = -\frac{3}{8\pi^2 k^2} \int_0^{k_F} dq q \{ & 2g_\rho^2 \hat{Q}\Phi_\rho + 2\left(\frac{f_\rho}{2M}\right)^2 [kq\hat{Q}\Theta_\rho - \hat{Q}(k^2 + q^2 - \frac{1}{2}m_\rho^2)\Phi_\rho] \\ & - 3g_\rho \left(\frac{f_\rho}{2M}\right) (k\hat{M}\Theta_\rho - 2q\hat{M}\Phi_\rho) \} \end{aligned} \quad (A.6)$$

Appendix B. Fock Term Expressions

The quantities X and Y of eq.[17] can be written as the sum of contributions coming from different mesons . In the following , we often need the reduced matrix elements of the tensorial operators $Y_L^m(\hat{r})$ and

$$T_{JL}^M \equiv \sum_{mk} \langle L1mk | JM \rangle Y_L^m(\hat{r}) \sigma^k$$

They are given as follow:

$$\begin{aligned} \langle a || Y_L || b \rangle = & \begin{cases} (4\pi)^{-\frac{1}{2}} \hat{j}_a \hat{j}_b \hat{L} (-1)^{j_b - L - \frac{1}{2}} \begin{pmatrix} j_a & j_b & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} & \text{if } l_a + l_b + L \text{ is even} \\ 0 & \text{if } l_a + l_b + L \text{ is odd} \end{cases} \\ \langle a || T_{JL} || b \rangle = & \left(\frac{6}{4\pi}\right)^{\frac{1}{2}} (-1)^{l_a} \hat{j}_a \hat{j}_b \hat{l}_a \hat{l}_b \hat{J} \hat{L} \begin{pmatrix} l_a & L & l_b \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} j_a & j_b & J \\ l_a & l_b & L \\ \frac{1}{2} & \frac{1}{2} & 1 \end{Bmatrix}, \end{aligned} \quad (B.1)$$

where $\hat{J} = \sqrt{2J+1}$.

The contribution coming from scalar meson σ is determined to be

$$\begin{aligned} \begin{pmatrix} -X^\sigma(r) \\ Y^\sigma(r) \end{pmatrix} &= g_\sigma(\rho_B(r)) m_\sigma \hat{j}_a^{-2} \sum_b \delta_{q_a q_b} \begin{pmatrix} F_b(r) \\ G_b(r) \end{pmatrix} \sum_L |< a || Y_L || b >|^2 \\ &\times \int_0^\infty g_\sigma(\rho_B(r')) [G_a G_b - F_a F_b]_{r'} \tilde{I}_L(m_\sigma r_<) \tilde{K}_L(m_\sigma r_>) dr' , \end{aligned} \quad (B.2)$$

the sum over b running over occupied states .

The expressions for the vector meson ω are splitted into timelike and spacelike parts , due to the respective γ_0 and γ coupling. The time component is

$$\begin{aligned} \begin{pmatrix} -X_0^\omega(r) \\ Y_0^\omega(r) \end{pmatrix} &= g_\omega(\rho_B(r)) m_\omega \hat{j}_a^{-2} \sum_b \delta_{q_a q_b} \begin{pmatrix} F_b(r) \\ -G_b(r) \end{pmatrix} \sum_L |< a || Y_L || b >|^2 \\ &\times \int_0^\infty g_\omega(\rho_B(r')) [G_a G_b + F_a F_b]_{r'} \tilde{I}_L(m_\omega r_<) \tilde{K}_L(m_\omega r_>) dr' , \end{aligned} \quad (B.3)$$

The space component is

$$\begin{aligned} \begin{pmatrix} -X^\omega(r) \\ Y^\omega(r) \end{pmatrix} &= -\frac{g_\omega(\rho_B(r))}{4\pi} m_\omega \sum_{b,L} \delta_{q_a q_b} (2j_b + 1)(2L + 1) \begin{pmatrix} G_b(r) \\ -F_b(r) \end{pmatrix} \\ &\times \int_0^\infty g_\omega(\rho_B(r')) \left\{ \begin{pmatrix} G_a F_b \\ -F_a G_b \end{pmatrix} \begin{pmatrix} j_a & j_b & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \right. \\ &+ \begin{pmatrix} F_a G_b \\ G_a F_b \end{pmatrix} \left[2 \begin{pmatrix} l_a & L & l'_b \\ 0 & 0 & 0 \end{pmatrix}^2 - \begin{pmatrix} j_a & j_b & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \right] \Big\} \\ &\times \tilde{I}_L(m_\omega r_<) \tilde{K}_L(m_\omega r_>) dr' \end{aligned} \quad (B.4)$$

where $a' = (q_a, n_a, l'_a, j_a)$ with $l'_a = 2j_a - l_a$.

The contribution comes from isovector pseudoscalar meson π with pseudovector coupling is as follows

$$\begin{aligned} \begin{pmatrix} -X^\pi \\ Y^\pi \end{pmatrix} &= f_\pi^2 \hat{j}_a^{-2} \sum_b (2 - \delta_{q_a q_b}) \left\{ \frac{j_a^2 j_b^2}{8\pi} \frac{(G_a G_b + F_a F_b)_r}{m_\pi^2 r^2} \begin{pmatrix} -F_b(r) \\ G_b(r) \end{pmatrix} \right. \\ &- m_\pi \sum_L \hat{L}^{-4} |< a || Y_L || b' >|^2 \sum_{L_1, L_2} \begin{pmatrix} F_b(r) [\kappa_{ab} - \alpha(L_1)] \\ G_b(r) [\kappa_{ab} + \alpha(L_1)] \end{pmatrix} i^{L_2 - l_1} \\ &\times \int_0^\infty [\{\kappa_{ab} + \alpha(L_2)\} G_a G_b - \{\kappa_{ab} - \alpha(L_2)\} F_a F_b]_{r'} R_{L_1, L_2}(m_\pi r, m_\pi r') dr' \Big\} . \end{aligned} \quad (B.5)$$

where we introduced the notation

$$\begin{aligned}\kappa_{ab} &= \kappa_a + \kappa_b, \\ \alpha(L_1) &= \begin{cases} -L & \text{if } L_1 = L - 1, \\ L + 1 & \text{if } L_1 = L + 1, \end{cases} \\ R_{L_1 L_2}(mr, mr') &= \tilde{I}_{L_1}(mr) \tilde{K}_{L_2}(mr') \theta(r' - r) + \tilde{K}_{L_1}(mr) \tilde{I}_{L_2}(mr') \theta(r - r').\end{aligned}$$

The L_1 and L_2 can only take two values $L + 1$ or $L - 1$.

The vector part of the ρ NN coupling in our Lagrangian gives $X(r)$ and $Y(r)$ which are formally identical to those of the ω meson, except for the isospin factor $\delta_{q_a q_b}$ replaced by $2 - \delta_{q_a q_b}$ and m_ω , $g_\omega(\rho_B(r))$ replaced by m_ρ , g_ρ . The tensor term gives rise to two types of contributions. They are proportional to f_ρ^2 and $f_\rho g_\rho$, and they are denoted, respectively, by $(X^{(T)}, Y^{(T)})$ and $(X^{(VT)}, Y^{(VT)})$. Furthermore, they can be split into timelike and spacelike component.

The time component of $X^{(T)}$ and $Y^{(T)}$ is written as

$$\begin{aligned}\begin{pmatrix} -X_0^{(T)}(r) \\ Y_0^{(T)}(r) \end{pmatrix} &= -\left(\frac{f_\rho}{2M}\right)^2 m_\rho^2 j_a^{-2} \sum_b (2 - \delta_{q_a q_b}) \left\{ \frac{j_a^2 j_b^2 (G_a F_b + F_a G_b)_r}{8\pi m_\rho^2 r^2} \begin{pmatrix} -G_b(r) \\ F_b(r) \end{pmatrix} \right. \\ &\quad - m_\rho \sum_L \hat{L}^{-4} |< a || Y_L || b >|^2 \sum_{L_1, L_2} \begin{pmatrix} G_b(r) [\tilde{\kappa}_{ab} - \alpha(L_1)] \\ F_b(r) [\tilde{\kappa}_{ab} + \alpha(L_1)] \end{pmatrix} i^{L_2 - L_1} \\ &\quad \times \int_0^\infty [\{\tilde{\kappa}_{ab} + \alpha(L_2)\} G_a F_b - \{\tilde{\kappa}_{ab} - \alpha(L_2)\} F_a G_b]_{r'} R_{L_1, L_2}(m_\rho r, m_\rho r') dr' \Big\}. \end{aligned} \quad (B.6)$$

where $\tilde{\kappa}_{ab} = \kappa_a - \kappa_b$.

The space component of $X^{(T)}$ and $Y^{(T)}$ is

$$\begin{aligned}\begin{pmatrix} -X^{(T)}(r) \\ Y^{(T)}(r) \end{pmatrix} &= 6 \left(\frac{f_\rho}{2M}\right)^2 m_\rho^2 j_a^{-2} \sum_b (2 - \delta_{q_a q_b}) \begin{pmatrix} -F_b(r) \\ G_b(r) \end{pmatrix} \sum_{LJ L_1 L_2} f_{LJ}(L_1) f_{LJ}(L_2) \\ &\quad \times \begin{pmatrix} < a' || T_{JL_1} || b' > \\ < a || T_{JL_1} || b > \end{pmatrix} \int_0^\infty [< a || T_{JL_2} || b > G_a G_b - < a' || T_{JL_2} || b' > F_a F_b]_{r'} \\ &\quad [m_\rho R_{L_1 L_2}(m_\rho r, m_\rho r') - \frac{\delta(r - r')}{m_\rho^2 r'^2}] dr', \end{aligned} \quad (B.7)$$

where we have introduced

$$f_{LJ}(L_1) = \hat{L} \hat{L}_1 \begin{pmatrix} L_1 & L & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} L_1 & L & 1 \\ 1 & 1 & J \end{Bmatrix}$$

The time components of VT contributions are

$$\begin{aligned}
\begin{pmatrix} -X_0^{(VT)}(r) \\ Y_0^{(VT)}(r) \end{pmatrix} &= \left(\frac{g_\rho f_\rho}{2M} \right) m_\rho^2 \hat{j}_a^{-2} \sum_b (2 - \delta_{q_a q_b}) \sum_{LL_1} (-1)^{L_1} \hat{L}_1 \begin{pmatrix} L_1 & L & 1 \\ 0 & 0 & 0 \end{pmatrix} \\
&\{ \begin{pmatrix} -G_b(r) < a' || T_{LL_1} || b > \\ F_b(r) < a || T_{LL_1} || b' > \end{pmatrix} \\
&\times \int_0^\infty [< a || Y_L || b > G_a G_b + < a' || Y_L || b' > F_a F_b]_{r'} S_{LL_1}(r, r') dr' \quad (B.8) \\
&+ \begin{pmatrix} -F_b(r) < a' || Y_L || b' > \\ G_b(r) < a || Y_L || b > \end{pmatrix} \\
&\times \int_0^\infty [< a || T_{LL_1} || b' > G_a F_b + < a' || T_{LL_1} || b > F_a G_b]_{r'} S_{LL_1}(r', r) dr' \}.
\end{aligned}$$

The space component of VT contribution are

$$\begin{aligned}
\begin{pmatrix} -X^{(VT)}(r) \\ Y^{(VT)}(r) \end{pmatrix} &= -\sqrt{6} \left(\frac{g_\rho f_\rho}{2M} \right) m_\rho^2 \hat{j}_a^{-2} \sum_b (2 - \delta_{q_a q_b}) \sum_{JLL_1} (-1)^J f_{LJ}(L_1) \\
&\{ \begin{pmatrix} F_b(r) < a' || T_{JL_1} || b' > \\ G_b(r) < a || T_{JL_1} || b > \end{pmatrix} \\
&\times \int_0^\infty [< a || T_{JL} || b' > G_a F_b - < a' || T_{JL} || b > F_a G_b]_{r'} S_{LL_1}(r, r') dr' \quad (B.9) \\
&+ \begin{pmatrix} G_b(r) < a' || T_{JL} || b > \\ F_b(r) < a || T_{JL} || b' > \end{pmatrix} \\
&\times \int_0^\infty [< a || T_{JL_1} || b > G_a G_b - < a' || T_{JL_1} || b' > F_a F_b]_{r'} S_{LL_1}(r, r') dr' \}.
\end{aligned}$$

where

$$S_{LL_1}(r, r') = \tilde{I}_{L_1}(m_\rho r) \tilde{K}_L(m_\rho r') \theta(m_\rho(r' - r)) - \tilde{I}_L(m_\rho r') \tilde{K}_{L_1}(m_\rho r) \theta(m_\rho(r - r')).$$

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Figure captions

Figure 1. Scalar and vector potentials U_s and U_0 as functions of the density in nuclear matter. The circles are the DBHF results using Bonn A potential[1]. The curves are obtained in terms of interpolations and extrapolations.

Figure 2. Density dependent coupling constants of σ and ω , g_σ and g_ω . They are deduced by reproducing the scalar and vector potentials of the DBHF results at each density from RMF analyses (a) and RHF analyses (b). The solid and dotted curves in (b) are corresponding to the cases of $\sigma + \omega$ and $\sigma + \omega + \pi + \rho$ in RHF. The circles on curves refer to the density points of the DBHF results.

Figure 3. Charge density distribution of various nuclei . The curves are the results of RDH (dashed one), RDHF with $\sigma + \omega$ only (dash-dotted) and RDHF with $\sigma + \omega + \pi + \rho$ (solid for 3C and dotted for 3A) and RHF with $\sigma + \omega + \pi + \rho$ (dence-dotted one) .

Figure 4. Binding energy versus $1/r_{ch}$ for ^{16}O and ^{40}Ca . The dash-dotted lines is taken from the work of Kümmel et al. [9]. The dotted and dashed lines are obtained by RHF3 with changing σ meson mass for $K_F = 1.4 \text{ fm}^{-1}$ (1) and $K_F = 1.3 \text{ fm}^{-1}$ (2) , respectively . The solid lines are the results of RDHF3C . The experimental data are displayed by a star .

Figure 5. Difference between charge densities of ^{40}Ca and ^{48}Ca multiplied by r^2 . The dashed curve corresponds to the RDH , the dotted curve to the RDHF1 . The solid line corresponds to the RDHF3C . The shaded area indicates the experimental data.

Figure 6. Same as Fig.4 for difference between neutron densities of ^{48}Ca and ^{40}Ca .

Figure 7. Δ_{np} versus the asymmetry parameter for various cases and the DBHF approach taken from ref.[20]